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Molecular Dynamical Method for Analyzing the Bio Molecular particle at the neighbor of the Biological Membrane.

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あらまし We introduce a mathematical method for predicting micro hydrodynamical behavior of one bio molecules at very narrow space that has been proposed by Ganatos (1982). One solution was an infinite series in spherical co-ordinates which have planar symmetry about the plane y=0 which is perpendicular to the longitudinal axis of the cylinder coordinate. This solution vanishes as r approaches to infinite. Another solution was a double Fourier integral in rectangular co-ordinates which produce finite velocities in the flow filed. The basic form of them were composed of Legendre spherical function, modified Bessel functions. The coefficients of these solutions were set to satisfy the no-slip boundary conditions on both infinite confining walls simultaneously for an arbitrary disturbance representing a sphere of unspecified size position d velocity. The present method will be available for predicting bio molecular particle at narrow biological space.

和文キーワード Microhydro dynamics. Cylindrical coordinate. Spherical Coordinate. Lamb. Legendre function.

有機生体膜近傍での生体粒子の動力学的解析法

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Abstract

細胞間などの狭い生体空間内を運動する生体分子の微小力学的挙動を推定する目的て Ganatos 1980が提唱した方法を紹介する。運動は並進、回転運動および１個の球周囲を通過する流れの３成分に分離して記述された。系は球粒子が通過する領域を円柱座標系で、球自身は球座表系で記述した。両系とも原点は球の中心とし、原点が必ずしも円柱の中心軸には一致しない一般的な場合を想定した。解析解はまず無限遠でゼロ、円管垂直軸周囲で平面対照な球座標系での無限級数でLamb の特別な形式である。 もう一つの解は流れの場に任意の箇所で有限な速度生み出す、直交座標系における解である。 解はレジェンドレ陪球関数、 変形ベッセル関数、双曲線関数の複雑な有限積分形式で与えられた。級数の係数は境界条件によって決定された。 本研究を発展させることにより、微小動力学的な生体空間における生体分子の挙動を推定することが可能である。

英文 key words 微小力学的挙動. 円柱座標系. 球座表系. Lamb. レジェンドレ陪関数、 変形ベッセル関数.

1. Introduction.

We introduce an interaction theoretical analysis of a bio molecular particle that traveling between narrow inter cellular space. The original approach was founded by Ganatos. The present paper follows from previous our technical report in this series. (IEICE.Tech. NC 20001-1. p1-p7). The Vw is the velocity in rectangular co-ordinates which produce finite velocities in flow field that is given

$$V_w = u_w i + v_w j + w_w k \qquad (2.7)$$

$$u_w = \int_0^\infty \int_0^\infty D_1(\alpha,\beta,z)\cos\alpha x\cos\beta y\, d\alpha\, d\beta, \qquad (2.8a)$$

$$v_w = \int_0^\infty \int_0^\infty D_2(\alpha,\beta,z)\sin\alpha x\sin\beta y\, d\alpha\, d\beta, \qquad (2.8b)$$

$$w_w = \int_0^\infty \int_0^\infty D_3(\alpha,\beta,z)\sin\alpha x\cos\beta y\, d\alpha\, d\beta, \qquad (2.8c)$$

$$D_1(\alpha,\beta,z) = \left[A^*\left(1+\frac{a^2}{k}z\right) - A^{**}\frac{\alpha\beta}{k}z - A^{***}\alpha z\right]e^{kz} + \left[B^*\left(1-\frac{a^2}{k}z\right) + B^{**}\frac{\alpha\beta}{k}z - B^{***}\alpha z\right]e^{-kz} \qquad (2.9a)$$

$$D_2(\alpha,\beta,z) = \left[-A^*\frac{\alpha\beta}{k}z + A^{**}\left(1+\frac{\beta^2}{k}z\right) + A^{***}\beta z\right]e^{kz} + \left[B^*\frac{\alpha\beta}{k}z + B^{**}\left(1-\frac{\beta^2}{k}z\right) + B^{***}\beta z\right]e^{-kz} \qquad (2.9b)$$

$$D_3(\alpha,\beta,z) = \left[A^*\alpha z - A^{**}\beta z + A^{***}(1-kz)\right]e^{kz} + \left[B^*\alpha z - B^{**}\beta z + B^{***}(1+kz)\right]e^{-kz} \qquad (2.9c)$$

$$r = (x^2+y^2+z^2)^{\frac{1}{2}} \qquad \theta = \cos^{-1}\left|\frac{z}{(x^2+y^2+z^2)^{\frac{1}{2}}}\right| \qquad (2.10c)$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \qquad (2.10c)$$

$$s = \cos\phi \quad ,t = \sin\phi \qquad \cos\theta = \frac{z}{r} \;,\sin\theta = \frac{1}{r}$$

$$\left.\begin{aligned}
&\int_0^\infty\int_0^\infty D_1(\alpha,\beta,z_i)\cos\alpha x\cos\beta y\, d\alpha\, d\beta \\
&= -\sum_{n=1}^\infty\left[A_nA_n'(x,y,z_i) + B_nB_n'(x,y,z_i) + C_nC_n'(x,y,z_i)\right] \\
&\int_0^\infty\int_0^\infty D_2(\alpha,\beta,z_i)\sin\alpha x\sin\beta y\, d\alpha\, d\beta \\
&= -\sum_{n=1}^\infty\left[A_nA_n''(x,y,z_i) + B_nB_n''(x,y,z_i) + C_nC_n''(x,y,z_i)\right] \\
&\int_0^\infty\int_0^\infty D_3(\alpha,\beta,z_i)\sin\alpha x\cos\beta y\, d\alpha\, d\beta \\
&= -\sum_{n=1}^\infty\left[A_nA_n'''(x,y,z_i) + B_nB_n'''(x,y,z_i) + C_nC_n'''(x,y,z_i)\right]
\end{aligned}\right\} i = 1,2, \qquad (2.11)$$

$$\left.\begin{aligned}
D_1(\alpha,\beta,z_i) &= -\frac{4}{\pi^2}\int_0^\infty\int_0^\infty\left\{\sum_{n=1}^\infty\left[A_nA_n'(s,t,z_i) + B_nB_n'(s,t,z_i) + C_nC_n'(s,t,z_i)\right]\right\}\cos\alpha s\cos\beta t\, ds\, dt, \\
D_2(\alpha,\beta,z_i) &= -\frac{4}{\pi^2}\int_0^\infty\int_0^\infty\left\{\sum_{n=1}^\infty\left[A_nA_n''(s,t,z_i) + B_nB_n''(s,t,z_i) + C_nC_n''(s,t,z_i)\right]\right\}\sin\alpha s\sin\beta t\, ds\, dt, \\
D_3(\alpha,\beta,z_i) &= -\frac{4}{\pi^2}\int_0^\infty\int_0^\infty\left\{\sum_{n=1}^\infty\left[A_nA_n'''(s,t,z_i) + B_nB_n'''(s,t,z_i) + C_nC_n'''(s,t,z_i)\right]\right\}\sin\alpha s\cos\beta t\, ds\, dt,
\end{aligned}\right\} i = 1,2 \qquad (2.12)$$

$$F_4(\alpha,\beta,z_i,n,m) = \int_0^\infty\int_0^\infty \frac{s}{(s^2+t^2+z_i^2)^{\frac{1}{2}(n+1)}} P_n^m\left|\frac{z_i}{(s^2+t^2+z_i^2)^{\frac{1}{2}}}\right| \sin\alpha s\cos\beta t\, ds\, dt$$

$$= \frac{\pi}{2}\alpha|z_i|^3 \sum_{q=0}^{\left[\frac{1}{2}n\right]} S_{nmq}(z_i)(k|z_i|)^{n-q-\frac{3}{2}} K_{n-q-\frac{3}{2}}(k|z_i|), \qquad (2.14d)$$

$$S_{nmq}(z_i) = \frac{(2/\pi)^{\frac{1}{2}}}{(-2)^q q!(n-2q-m)! z_i^{n+m}} \qquad (2.15) \qquad z_i = -b = c$$

$$\left.\begin{aligned}
D_1(\alpha,\beta,z_i) &= \sum_{n=1}^\infty\left[A_nA_n^*(\alpha,\beta,z_i) + B_nB_n^*(\alpha,\beta,z_i) + C_nX_n^*(\alpha,\beta,z_i)\right] \\
D_2(\alpha,\beta,z_i) &= \sum_{n=1}^\infty\left[A_nA_n^{**}(\alpha,\beta,z_i) + B_nB_n^{**}(\alpha,\beta,z_i) + C_nX_n^{**}(\alpha,\beta,z_i)\right] \\
D_3(\alpha,\beta,z_i) &= \sum_{n=1}^\infty\left[A_nA_n^{***}(\alpha,\beta,z_i) + B_nB_n^{***}(\alpha,\beta,z_i) + C_nX_n^{***}(\alpha,\beta,z_i)\right]
\end{aligned}\right\} i = 1,2$$

$$D_1(\alpha,\beta,z) = G_5(\eta)D_1(\alpha,\beta,-b) - G_5(\sigma)D_1(\alpha,\beta,c) + G_6(\sigma,\eta)\frac{\alpha}{k^2}\left[\alpha D_1(\alpha,\beta,-b) - \beta D_2(\alpha,\beta,-b)\right] - G_6(\eta,\sigma)\frac{\alpha}{k^2}\left[\alpha D_1(\alpha,\beta,c) - \beta D_2(\alpha,\beta,c)\right] + G_1(\sigma,\eta)\frac{\alpha}{k}D_3(\alpha,\beta,-b) - G_1(\eta,\sigma)\frac{\alpha}{k}D_3(\alpha,\beta,c), \qquad (2.16)\ (2.17a)$$

$$D_2(\alpha,\beta,z) = -G_6(\sigma,\eta)\frac{\alpha\beta}{k^2}\left[D_1(\alpha,\beta,-b) + \frac{\alpha}{\beta}D_2(\alpha,\beta,-b)\right] + G_6(\eta,\sigma)\frac{\alpha\beta}{k^2}\left[D_1(\alpha,\beta,c) + \frac{\alpha}{\beta}D_2(\alpha,\beta,c)\right] + G_5(\sigma,\eta)D_2(\alpha,\beta,-b) - G_5(\eta,\sigma)D_2(\alpha,\beta,c) - G_1(\sigma,\eta)\frac{\beta}{k}D_3(\alpha,\beta,-b) + G_1(\eta,\sigma)\frac{\beta}{k}D_3(\alpha,\beta,c), \qquad (2.17b)$$

$$D_3(\alpha,\beta,z) = G_2(\sigma,\eta)\left[\frac{\alpha}{\kappa}D_1(\alpha,\beta,-b) - \frac{\beta}{\kappa}D_2(\alpha,\beta,-b)\right] - G_2(\eta,\sigma)\left[\frac{\alpha}{\kappa}D_1(\alpha,\beta,c) - \frac{\beta}{\kappa}D_2(\alpha,\beta,c)\right] + G_4(\sigma,\eta)D_3(\alpha,\beta,-b) - G_4(\eta,\sigma)D_3(\alpha,\beta,c), \qquad (2.17c)$$

$$G_{1,2}(\mu,\nu) = 4\tau\mu\nu\left[\frac{\sinh\mu}{\mu} \pm \frac{\sinh\tau}{\tau}\frac{\sinh\nu}{\nu}\right]\Big/\delta_2 \qquad (2.18a)$$

$$G_{3,4}(\mu,\nu) = 4\tau\left\{\nu\left[\cosh\mu - \frac{\sinh\tau}{\tau}\frac{\sinh\nu}{\nu}\right] \pm \mu\left[\frac{\sinh\mu}{\mu} - \frac{\sinh\tau}{\tau}\cosh\nu\right]\right\}\Big/\delta_2 \qquad (2.18b)$$

$$G_5(\mu) = (-2\sinh\mu)/\delta_1 \qquad (2.18c)$$

$$G_6(\mu,\nu) = 8\tau^2\left\{\mu\frac{\sinh\tau}{\tau}\left[\frac{\sinh\mu}{\mu} - \frac{\sinh\tau}{\tau}\cosh\nu\right] + \nu\left[\frac{\sinh\tau}{\tau}\cosh\mu - \frac{\sinh\nu}{\nu}\right]\right\}\Big/\delta_1\delta_2 \qquad (2.18d)$$

In the above expressions, subscripts 1,3 and 2,4 refer to the + and - Signs on the right sides, μ and ν are dummy variables and

$$\delta_1 = 2\sinh\tau, \quad \delta_2 = 4\left[\sinh^2\tau - \tau^2\right] \qquad (2.19a,b)$$

$$\sigma = \kappa(z+b), \quad \eta = \kappa(z-c) \qquad (2.20a,b)$$

$$\tau = \kappa(b+c) \qquad (2.20c)$$

The double integrals required in (2,8) can not be preformed analytically. Hence, substituting

$$\alpha = \kappa\cos\gamma, \qquad \beta = \kappa\sin\gamma \qquad (2.21a,b)$$

transforms (2.8) into

$$u_w = \int_0^\infty\int_0^{\frac{1}{2}\pi}\kappa D_1(\kappa,\gamma,z)\cos(\kappa x\cos\gamma)\cos(\kappa y\sin\gamma)d\gamma\, d\kappa \qquad (2.22a)$$

$$v_w = \int_0^\infty\int_0^{\frac{1}{2}\pi}\kappa D_2(\kappa,\gamma,z)\sin(\kappa x\cos\gamma)\sin(\kappa y\sin\gamma)d\gamma\, d\kappa \qquad (2.22b)$$

$$w_w = \int_0^\infty\int_0^{\frac{1}{2}\pi}\kappa D_3(\kappa,\gamma,z)\sin(\kappa x\cos\gamma)\cos(\kappa y\sin\gamma)d\gamma\, d\kappa \qquad (2.22c)$$

where D1, D2 and D3 are functions of κ γ. The inner integral with respect to γ is obtined anakytically by Fourier Transformation.

V = u i + v j + w k ----(2,23)

us =voo + Σ[An {An'(x,y,z) +*An'(x,y,z)*} + Bn {Bn'(x,y,z) +*Bn'(x,y,z)*} + Cn {Cn'(x,y,z) +*Cn'(x,y,z)*}] ---(2-24a)

vs = Σ[An {An''(x,y,z) +*An''(x,y,z)*} + Bn {Bn''(x,y,z) +*Bn''(x,y,z)*} +Cn {Cn''(x,y,z) +*Cn''(x,y,z)*}] ---(2-24b)

ws= Σ[An {An'''(x,y,z) +*An'''(x,y,z)*}+ Bn {Bn'''(x,y,z) +*Bn'';(x,y,z)*}+Cn {Cn'''(x,y,z) +*Cn'''(x,y,z)*}] --(2-24b)

5. References

1. Ganatos P. J.F.M. vol 99. pp 755-783. 1980.
2. Ganatos, P. J.F.M. vol 84. pp 79-111. 1978.

I. Induction of (2. 24)

1.The velocity field

$V = V_\omega + V_s + V_\infty$ (2, 2)

$V_s = u_s\hat{i} + v_s\hat{j} + w_s\hat{k}$ (2, 4)

$V_\omega = u_\omega\hat{i} + v_\omega\hat{j} + w_\omega\hat{k}$ (2, 7)

$= u_s\hat{i} + v_s\hat{j} + w_s\hat{k} + u_\omega\hat{i} + v_\omega\hat{j} + w_\omega\hat{k} + V_\infty$

The first three terms are derived from eq(2, 5) .

For the second three setting in the equ (2, 8) as

$\alpha = \kappa \cdot \cos\gamma \qquad \beta = \kappa \cdot \sin\gamma$

then, in terms of equation (2, 22)

$$= \hat{i}\sum_{n=1}^{\infty}\left[A_n \cdot A_n' + B_n \cdot B_n' + C_n \cdot C_n'\right]$$
$$+ \hat{j}\sum_{n=1}^{\infty}\left[A_n \cdot A_n'' + B_n \cdot B_n'' + C_n \cdot C_n''\right]$$
$$+ \hat{k}\sum_{n=1}^{\infty}\left[A_n \cdot A_n''' + B_n \cdot B_n''' + C_n \cdot C_n'''\right]$$
$$+ \hat{i}\int_0^\infty\int_0^{\pi/2} \kappa D_1(\kappa,\gamma,z)\cos(\kappa x\cos\gamma)\cos(\kappa y\sin\gamma)d\gamma d\kappa$$
$$+ \hat{j}\int_0^\infty\int_0^{\pi/2} \kappa D_2(\kappa,\gamma,z)\sin(\kappa x\cos\gamma)\sin(\kappa y\sin\gamma)d\gamma d\kappa$$
$$+ \hat{k}\int_0^\infty\int_0^{\pi/2} \kappa D_3(\kappa,\gamma,z)\sin(\kappa x\cos\gamma)\cos(\kappa y\sin\gamma)d\gamma d\kappa$$

Here, for simplicity

$g_1(\kappa,\gamma) = \cos(\kappa\chi\cos\gamma)\cdot\cos(\kappa y\sin\gamma)$

$g_2(\kappa,\gamma) = \sin(\kappa\chi\cos\gamma)\cdot\sin(\kappa y\sin\gamma)$

$g_3(\kappa,\gamma) = \sin(\kappa\chi\cos\gamma)\cdot\cos(\kappa y\sin\gamma)$

We compute only the last three terms

For $D_1(\kappa,\gamma,z) \quad D_2(\kappa,\gamma,z) \quad D_3(\kappa,\gamma,z)$,

we substitute $(2{,}17,a,b,c)$

$$V_\omega = \hat{i}\int_0^\infty\int_0^{\pi/2} \kappa\cdot D_1(\kappa,\gamma,z)g_1(\kappa,\gamma)d\gamma\cdot d\kappa$$
$$+ \hat{j}\int_0^\infty\int_0^{\pi/2} \kappa\cdot D_2(\kappa,\gamma,z)g_2(\kappa,\gamma)d\gamma\cdot d\kappa$$
$$+ \hat{k}\int_0^\infty\int_0^{\pi/2} \kappa\cdot D_3(\kappa,\gamma,z)g_3(\kappa,\gamma)d\gamma\cdot d\kappa$$

For this, we substitute the terms in $(2{,}17,a,b,c)$ by

Converting $(\alpha,\beta)\rightarrow(\kappa,\gamma)$

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II] Expansion of V_ω

$V_\omega =$

$$\hat{i}\int_0^\infty\int_0^{\pi/2}\kappa\cdot\left[\left[G_5(\eta)\cdot D_1(\kappa,\gamma,-b) - G_5(\sigma)\cdot D_1(\kappa,\gamma,c)\right.\right.$$
$$+ G_6(\sigma,\eta)\cdot\kappa\cdot\cos\gamma/\kappa^2\left[\kappa\cos\gamma\cdot D_1(\kappa,\gamma,-b) - \kappa\cdot\sin\gamma\cdot D_2(\kappa,\gamma,-b)\right]$$
$$- G_6(\eta,\sigma)\cdot\kappa\cdot\cos\gamma/\kappa^2\left[\kappa\cos\gamma\cdot D_1(\kappa,\gamma,c) - \kappa\cdot\sin\gamma\cdot D_2(\kappa,\gamma,c)\right]$$
$$\left.\left.+ G_1(\sigma,\eta)\cdot\kappa\cdot\cos\gamma/\kappa\cdot D_3(\kappa,\lambda,-b) - G_1(\eta,\sigma)\cdot\kappa\cos\gamma/\kappa\cdot D_3(\kappa,\gamma,c)\right]\right]$$
$$\cdot g_1(\kappa,\gamma)d\gamma\cdot d\kappa$$
$$+\hat{j}\int_0^\infty\int_0^{\pi/2}\kappa\cdot\left[\left[-G_6(\sigma,\eta)\kappa^2\cos\gamma\cdot\sin\gamma/\kappa^2\cdot\left[D_1(\kappa,\gamma,-b)+\cos\gamma/\sin\gamma\cdot D_2(\kappa,\gamma,-b)\right]\right.\right.$$
$$+ G_6(\eta,\sigma)\cdot\kappa^2\cos\gamma\cdot\sin\gamma/\kappa^2\left[D_1(\kappa,\gamma,c) + \cos\gamma/\sin\gamma\cdot D_2(\kappa,\gamma,c)\right]$$
$$+ G_3(\sigma,\eta)\cdot D_2(\kappa,\gamma,-b) - G_3(\eta,\sigma)D_2(\kappa,\gamma,c)$$
$$\left.\left.- G_1(\sigma,\eta)\cdot\kappa\cdot\sin\gamma/\kappa\cdot D_3(\kappa,\gamma,-b) + G_1(\eta,\sigma)\cdot\kappa\sin\gamma/\kappa\cdot D_3(\kappa,\gamma,c)\right]\right]$$
$$\cdot g_2(\kappa,\gamma)d\gamma\cdot d\kappa$$
$$+\hat{k}\int_0^\infty\int_0^{\pi/2}\kappa\cdot\left[\left[G_2(\sigma,\eta)\cdot\left[\kappa\cos\gamma/\kappa\cdot D_1(\kappa,\gamma,-b) - \kappa\sin\gamma/\kappa\cdot D_2(\kappa,\gamma,-b)\right]\right.\right.$$
$$- G_2(\eta,\sigma)\cdot\left[\kappa\cos\gamma/\kappa\cdot D_1(\kappa,\gamma,c) - \kappa\sin\gamma/\kappa\cdot D_2(\kappa,\gamma,c)\right]$$
$$\left.\left.+ G_4(\sigma,\eta)\cdot D_3(\kappa,\gamma,-b) - G_4(\eta,\sigma)\cdot D_3(\kappa,\gamma,c)\right]\right]$$
$$\cdot g_3(\kappa,\gamma)d\gamma\cdot d\kappa$$

Rearrange these in terms of $G_n(\sigma,\eta)$

$$= \hat{i}\left[\left[\int_0^\infty\int_0^{\pi/2}\kappa\cdot G_5(\eta)\cdot D_1(\kappa,\gamma,-b)\cdot g_1(\kappa,\gamma)d\gamma d\kappa\right.\right.$$
$$-\int_0^\infty\int_0^{\pi/2}\kappa\cdot G_5(\sigma)\cdot D_1(\kappa,\gamma,c)\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$
$$+\int_0^\infty\int_0^{\pi/2}\kappa G_6(\sigma,\eta)\cdot\left[\cos^2\gamma\cdot D_1(\kappa,\gamma,-b) - \cos\gamma\cdot\sin\gamma\cdot D_2(\kappa,\gamma,-b)\right]\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$
$$-\int_0^\infty\int_0^{\pi/2}\kappa G_6(\eta,\sigma)\cdot\left[\cos^2\gamma\cdot D_1(\kappa,\gamma,c) - \cos\gamma\cdot\sin\gamma\cdot D_2(\kappa,\gamma,c)\right]\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$
$$+\int_0^\infty\int_0^{\pi/2}\kappa G_1(\sigma,\eta)\cdot\cos\gamma\cdot D_3(\kappa,\gamma,-b)\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$
$$\left.\left.-\int_0^\infty\int_0^{\pi/2}\kappa\cdot G_1(\eta,\sigma)\cos\gamma\cdot D_3(\kappa,\gamma,c)g_1(\kappa,\gamma)d\gamma d\kappa\right]\right]$$
$$+\hat{j}\left[\left[\int_0^\infty\int_0^{\pi/2}(-\kappa)\cdot G_6(\sigma,\eta)\cdot\left[\cos\gamma\cdot\sin\gamma\cdot D_1(\kappa,\gamma,-b) + \cos^2\gamma\cdot D_2(\kappa,\gamma,-b)\right]\right.\right.$$
$$+\int_0^\infty\int_0^{\pi/2}\kappa G_6(\eta,\sigma)\cdot\left[\cos\gamma\cdot\sin\gamma\cdot D_1(\kappa,\gamma,c) + \cos^2\gamma\cdot D_2(\kappa,\gamma,c)\right]\cdot g_2(\kappa,\gamma)d\gamma d\kappa$$
$$+\int_0^\infty\int_0^{\pi/2}\kappa\cdot G_3(\sigma,\eta)\cdot D_2(\kappa,\gamma,-b)\cdot g_2(\kappa,\gamma)d\gamma d\kappa$$
$$-\int_0^\infty\int_0^{\pi/2}\kappa\cdot G_3(\eta,\sigma)\cdot D_2(\kappa,\gamma,c)\cdot g_2(\kappa,\gamma)d\gamma d\kappa$$
$$-\int_0^\infty\int_0^{\pi/2}\kappa\cdot G_1(\sigma,\eta)\cdot\sin\gamma\cdot D_3(\kappa,\gamma,-b)\cdot g_2(\kappa,\gamma)d\gamma d\kappa$$
$$\left.\left.+\int_0^\infty\int_0^{\pi/2}\kappa\cdot G_1(\eta,\sigma)\sin\gamma\cdot D_3(\kappa,\gamma,c)g_2(\kappa,\gamma)d\gamma d\kappa\right]\right]$$
$$+\hat{k}\left[\left[\int_0^\infty\int_0^{\pi/2}\kappa\cdot G_2(\sigma,\eta)\cdot\left[\cos\gamma\cdot D_1(\kappa,\gamma,-b) - \sin\gamma\cdot D_2(\kappa,\gamma,-b)\right]g_3(\kappa,\gamma)d\gamma d\kappa\right.\right.$$
$$-\int_0^\infty\int_0^{\pi/2}\kappa G_2(\eta,\sigma)\cdot\left[\cos\gamma\cdot D_1(\kappa,\gamma,c) - \sin\gamma\cdot D_2(\kappa,\gamma,c)\right]\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$
$$+\int_0^\infty\int_0^{\pi/2}\kappa G_4(\sigma,\eta)\cdot D_3(\kappa,\gamma,-b)\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$
$$\left.\left.-\int_0^\infty\int_0^{\pi/2}\kappa\cdot G_4(\eta,\sigma)\cdot D_3(\kappa,\gamma,c)g_3(\kappa,\gamma)d\gamma d\kappa\right]\right]$$

where $D_{1,2,3}(\kappa,\gamma,-b) \quad D_{1,2,3}(\kappa,\gamma,c)$ are given by equation (2、16). As

$$\int_0^\infty\int_0^{\pi/2}\kappa G_n(\sigma,\eta)\cdot D_m(\kappa,\gamma,-b,c)\cdot g_l(\kappa,\gamma)d\gamma d\kappa$$
$$=\int_0^\infty\int_0^{\pi/2}\kappa G_n(\sigma,\eta)\cdot\sum_{n=1}\left[A_n\cdot A_n^\bullet(\kappa,\gamma,-b,c) + B_n\cdot B_n^\bullet(\kappa,\gamma,-b,c)\right.$$
$$\left.+ C_n\cdot C_n^\bullet(\kappa,\gamma,-b,c)\right]\cdot g_l(\kappa,\gamma)d\gamma d\kappa$$
$$=\sum_{n=1}A_n\int_0^\infty\int_0^{\pi/2}\kappa G_n(\sigma,\eta)\cdot A_n^\bullet(\kappa,\gamma,-b,c)\cdot g_l(\kappa,\gamma)d\gamma d\kappa$$
$$+\sum_{n=1}B_n\int_0^\infty\int_0^{\pi/2}\kappa G_n(\sigma,\eta)\cdot B_n^\bullet(\kappa,\gamma,-b,c)\cdot g_l(\kappa,\gamma)d\gamma d\kappa$$
$$+\sum_{n=1}C_n\int_0^\infty\int_0^{\pi/2}\kappa G_n(\sigma,\eta)\cdot C_n^\bullet(\kappa,\gamma,-b,c)\cdot g_l(\kappa,\gamma)d\gamma d\kappa$$

while $G_n(\sigma,\eta)$ is given by (2, 18)

$A_n^\bullet$、$B_n^\bullet$、$C_n^\bullet$ are given by $(A_1 \sim A_9)$. Hence

the integration about γ can be determined.

Since $F_m(\kappa,\gamma,-b,c,n,m)$ in A1 to A9 are determined by (2, 14)、 the nuclear function of the integrand is

$$\int_0^\infty\int_0^{\pi/2}F_l(\kappa,\gamma,-b,c,n,m)\cdot g_l(\kappa,\gamma)d\gamma d\kappa$$
$$=\int_0^\infty\int_0^{\pi/2}\sum S_{nmg}(\kappa,-b,c)\cdot K_{n-q-l}(\kappa,-b,c)\cdot g_l(\kappa,\gamma)d\gamma d\kappa$$

% from c－5, the series part can be expressed by $B_{n,m,j,l}(z_i = -b,c)$

$$=\int_0^\infty B_{n,m,j,l}(\kappa)\int_0^{\pi/2}g_l(\kappa,\gamma)d\gamma\cdot d\kappa$$

****

III] Integration of each term of V_ω

The terms are related to $\hat{k}$ and (2, 23)$V = \hat{i}u + \hat{j}v + \hat{k}w$

Hence, it leads to the form of (2, 24c)

The coefficients of A_n''' 、B_n''' 、C_n''' are

①$\hat{k}\cdot\int_0^\infty\int_0^{\pi/2}\kappa\ G_4(\eta,\sigma)\cdot D_3(\kappa,\gamma,c)\cdot g_3(\kappa,\gamma)d\gamma d\kappa$

from (2、16、c)

$$D_3(\kappa,\gamma,c)=\sum_{n=1}\left[A_n\cdot A_n'''(\kappa,\gamma,c)+B_n\cdot B_n'''(\kappa,\gamma,c)+C_n\cdot C_n'''(\kappa,\gamma,c)\right]$$

omitting $\hat{k}$ from after

$$=\sum_{n=1}^\infty A_n\int_0^\infty\int_0^{\pi/2}\kappa\ G_4(\eta,\sigma)\cdot A_n'''(\kappa,\gamma,c)\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$
$$+\sum_{n=1}^\infty B_n\int_0^\infty\int_0^{\pi/2}\kappa\ G_4(\eta,\sigma)\cdot B_n'''(\kappa,\gamma,c)\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$
$$+\sum_{n=1}^\infty C_n\int_0^\infty\int_0^{\pi/2}\kappa\ G_4(\eta,\sigma)\cdot C_n'''(\kappa,\gamma,c)\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$

in the integrand, A_n''' is given by $(A,7)$ B_n''' is given by A_8 and C_n''' by A_9. Thus setting $z_i=c$

$$=\sum_{n=1}^\infty A_n\int_0^\infty\int_0^{\pi/2}\kappa\ G_4(\eta,\sigma)\cdot(-4/\pi^2)\cdot[n\cdot(2n-1)c\cdot F_4(\kappa,\gamma,c,n,1)$$
$$-(n+1)(n-2)\cdot F_4(\kappa,\gamma,c,n-1,1)]\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$
$$+\sum_{n=1}^\infty B_n\int_0^\infty\int_0^{\pi/2}\kappa\ G_4(\eta,\sigma)\cdot 4/\pi^2\cdot n\cdot F_4(\kappa,\gamma,c,n+1,1)\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$
$$+\sum_{n=1}^\infty C_n\int_0^\infty\int_0^{\pi/2}\kappa\ G_4(\eta,\sigma)\cdot 4/\pi^2\cdot F_4(\kappa,\gamma,c,n,1)\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$

****

We compute individual terms omitting $(4/\pi^2)$

①—c: $\sum C_n$

$$\int_0^\infty\int_0^{\pi/2}\kappa\ G_4(\eta,\sigma)\cdot F_4(\kappa,\gamma,c,n,1)\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$

from (2、14、d)

$$F_4(\kappa,\gamma,c,n,1)=\pi/2\cdot\kappa\cos\gamma\cdot c^3\sum_{q=0}^{[n/2]}S_{n,1,q}(c)(\kappa c)^{n-q-3/2}K_{n-q-3/2(\kappa c)}$$

$$=G_4(\eta,\sigma)\int_0^\infty\int_0^{\pi/2}\kappa\cdot(\pi/2)\cdot\kappa\cos\gamma\cdot c^3\sum_{q=0}^{[n/2]}S_{n,1,q}(c)(\kappa c)^{n-q-3/2}K_{n-q-3/2(\kappa c)}\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$

$$=(\pi/2)\cdot G_4(\eta,\sigma)\cdot\sum_{q=0}^{[n/2]}\int_0^\infty\int_0^{\pi/2}S_{n,1,q}(c)\cdot\kappa^2\cdot c^3(\kappa c)^{n-q-3/2}K_{n-q-3/2(\kappa c)}\cdot\cos\gamma\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$

$$=(\pi/2)\cdot G_4(\eta,\sigma)\cdot\sum_{q=0}^{[n/2]}\int_0^\infty\int_0^{\pi/2}S_{n,1,q}(c)\cdot\frac{(\kappa c)^{n-q-3/2+3}}{\kappa}\cdot K_{n-q-3/2(\kappa c)}\cdot\cos\gamma\cdot\sin(\kappa\chi\cos\gamma)\cdot\cos(\kappa y\sin\gamma)d\gamma d\kappa$$

from the definition of C−5

$$B_{n,m,j,l}(z_i)=\sum_{q=0}^{[n/2]}S_{n,m,q}(z_i)(\kappa z_i)^{n-q+l-1/2}K_{n-q-j-1/2(\kappa z_i)}$$

setting $-3/2=-j-1/2$ $\therefore j=1$

$3/2=l-1/2$ $\therefore l=2$

we have $\sum_{q=0}^{[n/2]}S_{n,1,q}(c)(\kappa c)^{n-q+3/2}\cdot K_{n-q-3/2(\kappa c)}=B_{n,1,1,2}$

$$=\pi/2\cdot G_4(\eta,\sigma)\int_0^\infty\int_0^{\pi/2}B_{n,1,1,2}\cdot(1/\kappa)\cdot\cos\gamma\cdot\sin(\kappa\chi\cos\gamma)\cos(\kappa y\sin\gamma)d\gamma d\kappa$$

*The integration is generally

$$\int_0^{\pi/2}\cos\gamma\cdot\sin(a\cos\gamma)\cdot\cos(b\sin\gamma)d\gamma=\pi/2\cdot a\cdot J_1(u)/u$$
$$u=(a^2+b^2)^{1/2}$$

by putting

$a=\kappa\chi$ $b=\kappa y$

$$u=(\kappa^2\chi^2+\kappa^2y^2)^{1/2}=\kappa\cdot(\chi^2+y^2)^{1/2}=\kappa\cdot\rho$$

hence

$$\int_0^{\pi/2}\cos\gamma\cdot\sin(\kappa\chi\cos\gamma)\cdot\cos(\kappa y\sin\gamma)d\gamma=\pi/2\cdot\kappa\cdot\chi\cdot J_1(\kappa\rho)/(\kappa\rho)$$

thus

$$=\pi/2\cdot G_4(\eta,\sigma)\int_0^\infty B_{n,1,1,2}(1/\kappa)\cdot\pi/2\cdot\kappa\chi\cdot J_1(\kappa\rho)/(\kappa\rho)d\kappa$$
$$=(\pi/2)^2\cdot G_4(\eta,\sigma)\int_0^\infty B_{n,1,1,2}\cdot\chi/(\kappa\sigma)\cdot J_1(\kappa\rho)d\kappa$$
$$=(\pi/2)^2\cdot G_4(\eta,\sigma)\int_0^\infty H_{24}(c)d\kappa$$
$$\therefore\hat{k}\cdot G_4(\eta,\sigma)\cdot H_{24}$$

****

②·b: $\sum B_n$

$$\hat{k}\int_0^\infty\int_0^{\pi/2}\kappa\ G_4(\eta,\sigma)\cdot n\cdot F_4(\kappa,\gamma,c,n+1,1)\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$

from (2、14、d)

$$F_4(\kappa,\gamma,c,n+1,1)=\pi/2\cdot\kappa\cos\gamma\cdot c^3\sum_{q=0}^{[(n+1)/2]}S_{n+1,1,q}(c)(\kappa c)^{n+1-q-3/2}\cdot K_{n+1-q-3/2(\kappa c)}$$

hence

$$=\pi/2\cdot G_4(\eta,\sigma)\cdot n\cdot\int_0^\infty\int_0^{\pi/2}\kappa\cdot\kappa\cdot\cos\gamma\cdot c^3\sum_{q=0}^{[(n+1)/2]}S_{n+1,1,q}(c)(\kappa c)^{n+1-q-3/2}\cdot K_{n+1-q-3/2(\kappa c)}\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$

$$=\pi/2\cdot G_4(\eta,\sigma)\cdot n\cdot\sum_{q=0}^{[(n+1)/2]}\int_0^\infty\int_0^{\pi/2}S_{n+1,1,q}(c)\cdot\frac{(\kappa c)^{n-q-1/2+3}}{\kappa}K_{n-q-1/2(\kappa c)}\cdot\cos\gamma\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$

from the definition of C—5

$$B_{n+1,m,j,l}=\sum_{q=0}^{[(n+1)/2]}S_{n+1,m,q}(\kappa c)^{n-q+l+1/2}\cdot K_{n-q-j+1/2}$$

putting $-1/2=-j+1/2$ $\therefore j=1$

$5/2=l+1/2$ $\therefore l=2$

$$=\pi/2\cdot G_4(\eta,\sigma)\cdot n\int_0^\infty\int_0^{\pi/2}B_{n+1,1,1,2}\cdot(1/\kappa)\cdot g_3(\kappa,\gamma)\cdot\cos\gamma d\gamma d\kappa$$
$$=\pi/2\cdot G_4(\eta,\sigma)\cdot n\int_0^\infty\int_0^{\pi/2}B_{n+1,1,1,2}\cdot(1/\kappa)\cdot\cos\gamma\cdot\sin(\kappa\chi\cos\gamma)\cos(\kappa y\sin\gamma)d\gamma d\kappa$$
$$=\pi/2\cdot G_4(\eta,\sigma)\cdot n\int_0^\infty B_{n+1,1,1,2}(1/\kappa)\cdot\pi/2\cdot\kappa\cdot\chi\cdot J_1(\kappa\rho)/(\kappa\rho)d\kappa$$
$$=(\pi/2)^2\cdot G_4(\eta,\sigma)\int_0^\infty B_{n+1,1,1,2}\cdot n\chi/\kappa\rho\cdot J_1(\kappa\rho)d\kappa$$
$$=(\pi/2)^2\cdot G_4(\eta,\sigma)\int_0^\infty H_{22}(c)d\kappa$$
$$\therefore\hat{k}\cdot G_4(\eta,\sigma)\cdot H_{22}$$

****

②·a: A_n omitting $(-4/\pi^2)$

$$\hat{k}\int_0^\infty\int_0^{\pi/2}\kappa\ G_4(\eta,\sigma)\cdot n\cdot(2n-1)\cdot c\cdot F_4(\kappa,\gamma,c,n,1)\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$
$$-\int_0^\infty\int_0^{\pi/2}\kappa\ G_4(\eta,\sigma)\cdot(n+1)(n-2)\cdot F_4(\kappa,\gamma,c,n-1,1)\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$

from (2、14、d)

$$F_4(\kappa,\gamma,c,n,1)=\pi/2\cdot\kappa\cos\gamma\cdot c^3\sum_{q=0}^{[n/2]}S_{n,1,q}(c)(\kappa c)^{n-q-3/2}\cdot K_{n-q-3/2(\kappa c)}$$
$$F_4(\kappa,\gamma,c,n-1,1)=\pi/2\cdot\kappa\cos\gamma\cdot c^3\sum_{q=0}^{[(n-1)/2]}S_{n-1,1,q}(c)(\kappa c)^{n-1-q-3/2}\cdot K_{n-1-q-3/2(\kappa c)}$$
$$=\pi/2\cdot\kappa\cos\gamma\cdot c^3\sum_{q=0}^{[(n-1)/2]}S_{n-1,1,q}(c)(\kappa c)^{n-q-5/2}\cdot K_{n-q-5/2(\kappa c)}$$

$$=G_4(\eta,\sigma)\int_0^\infty\int_0^{\pi/2}n\cdot(2n-1)\cdot\kappa\cdot c\cdot(\pi/2)\cdot\kappa\cdot\cos\gamma\cdot c^3\sum_{q=0}^{[n/2]}S_{n,1,q}(c)(\kappa c)^{n-q-3/2}\cdot K_{n-q-3/2(\kappa c)}\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$
$$-G_4(\eta,\sigma)\int_0^\infty\int_0^{\pi/2}(n+1)\cdot(n-2)\cdot\kappa\cdot(\pi/2)\cdot\kappa\cdot\cos\gamma\cdot c^3\sum_{q=0}^{[(n-1)/2]}S_{n-1,1,q}(c)(\kappa c)^{n-q-5/2}\cdot K_{n-q-5/2(\kappa c)}\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$
$$=\pi/2\cdot G_4(\eta,\sigma)\int_0^\infty\int_0^{\pi/2}n\cdot(2n-1)\cdot\sum_{q=0}^{[n/2]}S_{n,1,q}(c)\kappa^2\cdot c^4(\kappa c)^{n-q-3/2}\cdot K_{n-q-3/2(\kappa c)}\cdot\cos\gamma\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$

$$-\pi/2\cdot G_4(\eta,\sigma)\int_0^\infty\int_0^{\pi/2}(n+1)\cdot(n-2)\cdot\sum_{q=0}^{[(n-1)/2]}S_{n-1,1,q}(c)\kappa^2\cdot c^3(\kappa c)^{n-q-5/2}\cdot K_{n-q-5/2(\kappa c)}$$
$$\cdot\cos\gamma\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$

$$=\pi/2\cdot G_4(\eta,\sigma)[$$
$$\int_0^\infty\int_0^{\pi/2}n\cdot(2n-1)\cdot\sum_{q=0}^{[n/2]}S_{n,1,q}(c)\frac{c(\kappa c)^{n-q-3/2+3}}{\kappa}\cdot K_{n-q-3/2(\kappa c)}\cdot\cos\gamma\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$
$$-\int_0^\infty\int_0^{\pi/2}(n+1)(n-2)\cdot\sum_{q=0}^{[(n-1)/2]}S_{n-1,1,q}(c)\frac{(\kappa c)^{n-q-5/2+3}}{\kappa}\cdot K_{n-q-5/2(\kappa c)}\cdot\cos\gamma\cdot g_3(\kappa,\gamma)d\gamma d\kappa]$$

by the definition of

$$B_{n,m,j,l}(c)=\sum_{q=0}^{[n/2]}S_{n,m,q}(c)(\kappa c)^{n-q+l-1/2}\cdot K_{n-q-j-1/2(\kappa c)}$$

putting $3/2=l-1/2,\ -3/2=-j-1/2\ \therefore l=2,j=1$

putting $1/2=l-1/2-1,\ -5/2=-j-1/2-1\ \therefore l=2,j=1$

$$=\pi/2\cdot G_4(\eta,\sigma)[$$
$$\int_0^\infty\int_0^{\pi/2}n\cdot(2n-1)\cdot B_{n,1,1,2}(c)\cdot c/\kappa\cdot\cos\gamma\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$
$$-\int_0^\infty\int_0^{\pi/2}(n+1)(n-2)\cdot B_{n-1,1,1,2}(c)\cdot 1/\kappa\cdot\cos\gamma\cdot g_3(\kappa,\gamma)d\gamma d\kappa]$$

$$*\int_0^{\pi/2}\cos\gamma\cdot g_3(\kappa,\gamma)d\gamma=\pi/2\cdot\chi/\rho\cdot J_1(\kappa\rho)$$

$$=(\pi/2)^2\cdot G_4(\eta,\sigma)[$$
$$\int_0^\infty n\cdot(2n-1)\cdot B_{n,1,1,2}(c)\cdot c/\kappa\cdot\chi/\rho\cdot J_1(\kappa\rho)d\kappa$$
$$-\int_0^\infty(n+1)(n-2)\cdot B_{n-1,1,1,2}(c)\cdot 1/\kappa\cdot\chi/\rho\cdot J_1(\kappa\rho)d\kappa]$$

$$=(\pi/2)^2\cdot G_4(\eta,\sigma)\cdot\int_0^\infty\chi/(\kappa\cdot\rho)\cdot J_1(\kappa\rho)\cdot[n\cdot(2n-1)\cdot c\cdot B_{n,1,1,2}(c)$$
$$-(n+1)(n-2)\cdot B_{n-1,1,1,2}(c)]d\kappa$$

$$=(\pi/2)^2\cdot\hat{k}\cdot G_4(\eta,\sigma)\int_0^\infty H_{20}(c)d\kappa$$

Associating them

$$\hat{k}\cdot\int_0^\infty\int_0^{\pi/2}\kappa\ G_4(\eta,\sigma)\cdot D_3(\kappa,\gamma,c)\cdot g_3(\kappa,\gamma)d\gamma d\kappa$$
$$=\sum_{n=1}A_n\cdot\left(-G_4(\eta,\sigma)\int_0^\infty H_{20}(c)d\kappa\right)$$
$$+\sum_{n=1}B_n\cdot G_4(\eta,\sigma)\int_0^\infty H_{22}(c)d\kappa$$
$$+\sum_{n=1}C_n\cdot G_4(\eta,\sigma)\int_0^\infty H_{24}(c)d\kappa$$

****

② $\hat{i}\int_0^\infty\int_0^{\pi/2}\kappa\ G_5(\sigma)\cdot D_1(\kappa,\gamma,c)\cdot g_1(\kappa,\gamma)d\gamma d\kappa$

Since, this term is timed by $\hat{i}$, the form leads to Equations (2, 23)と(2, 24, a) and A'_n 、B'_n 、C'_n

Contribute as components. From (2、16, a)

$$=\int_0^\infty\int_0^{\pi/2}\kappa G_5(\sigma)\cdot\sum_{n=1}[A_n\cdot A_n^*(\kappa,\gamma,c)+B_n\cdot B_n^*(\kappa,\gamma,c)+C_n\cdot C_n^*(\kappa,\gamma,c)]\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$
$$=\sum_{n=1}A_n\int_0^\infty\int_0^{\pi/2}\kappa\ G_5(\sigma)\cdot A_n^*(\kappa,\gamma,c)\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$
$$+\sum_{n=1}B_n\int_0^\infty\int_0^{\pi/2}\kappa\ G_5(\sigma)\cdot B_n^*(\kappa,\gamma,c)\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$
$$+\sum_{n=1}C_n\int_0^\infty\int_0^{\pi/2}\kappa\ G_5(\sigma)\cdot C_n^*(\kappa,\gamma,c)\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$

A_n^* 、B_n^* 、C_n^* are given by A_1,A_2,A_3

Putting $z_i=c$

$$=\sum_{n=1}A_n\int_0^\infty\int_0^{\pi/2}\kappa\ G_5(\sigma)\cdot(-4/\pi^2)\cdot g_1(\kappa,\gamma)$$
$$\cdot\{n\cdot(2n-1)F_2(\kappa,\gamma,c,n,1)+(n-2)/2\cdot[F_2(\kappa,\gamma,c,n-1,2)-F_2(\gamma,\kappa,c,n-1,2)]$$
$$-n\cdot(n+1)(n-2)/2\cdot F_1(\kappa,\gamma,c,n-1,0)\}d\gamma d\kappa$$

$$+\sum_{n=1}B_n\int_0^\infty\int_0^{\pi/2}\kappa\ G_5(\sigma)\cdot(-4/\pi^2)\cdot g_1(\kappa,\gamma)$$
$$\cdot\{-1/2\cdot[F_2(\kappa,\gamma,c,n+1,2)-F_2(\gamma,\kappa,c,n+1,2)]+n(n+1)/2\cdot F_1(\kappa,\gamma,c,n+1,0)\}d\gamma d\kappa$$
$$+\sum_{n=1}C_n\int_0^\infty\int_0^{\pi/2}\kappa\ G_5(\sigma)\cdot(-4/\pi^2)\cdot g_1(\kappa,\gamma)$$
$$\cdot\{1/2\cdot[F_2(\kappa,\gamma,c,n,2)-F_2(\gamma,\kappa,c,n,2)]+n(n+1)/2\cdot F_1(\kappa,\gamma,c,n,0)\}d\gamma d\kappa$$

****

②・C : about C_n $G_5(\sigma)\cdot(-4/\pi^2)/2$

$$\int_0^\infty\int_0^{\pi/2}\kappa\cdot F_2(\kappa,\gamma,c,n,2)\cdot g_1(\kappa,\gamma)d\gamma d\kappa-\int_0^\infty\int_0^{\pi/2}\kappa\cdot F_2(\gamma,\kappa,c,n,2)\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$
$$+n(n+1)\int_0^\infty\int_0^{\pi/2}\kappa\cdot F_1(\kappa,\gamma,c,n,0)\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$

****

②・C－1 : from (2, 14, b), putting $m=2$

$$F_2(\kappa,\gamma,c,n,2)=\pi/2\cdot c^3\left[\sum_{q=0}^{[n/2]}S_{n,2,q}(c)(\kappa c)^{n-q-3/2}\cdot K_{n-q-3/2(\kappa c)}\right.$$
$$\left.-\sum_{q=0}^{[n/2]}S_{n,2,q}(c)\cdot\kappa^2\cos\gamma^2\cdot c^2(\kappa c)^{n-q-5/2}\cdot K_{n-q-5/2(\kappa c)}\right]$$
$$=\pi/2\cdot\left[\sum_{q=0}^{[n/2]}S_{n,2,q}(c)\frac{(\kappa c)^{n-q-3/2+3}}{\kappa^3}\cdot K_{n-q-3/2(\kappa c)}\right.$$
$$\left.-\sum_{q=0}^{[n/2]}S_{n,2,q}(c)\cdot\kappa^2\cos^2\gamma\cdot c^5(\kappa c)^{n-q-5/2}\cdot K_{n-q-5/2(\kappa c)}\right]$$
$$=\pi/2\cdot\left[\sum_{q=0}^{[n/2]}S_{n,2,q}(c)\frac{(\kappa c)^{n-q+3/2}}{\kappa^3}\cdot K_{n-q-3/2(\kappa c)}\right.$$
$$\left.-\sum_{q=0}^{[n/2]}S_{n,2,q}(c)\cdot\frac{(\kappa c)^{n-q-5/2+5}}{\kappa^3}\cdot K_{n-q-5/2(\kappa c)}\cdot\cos^2\gamma\right]$$

from the definition of C－5

$$\sum_{q=0}^{[n/2]}S_{n,2,q}(c)(\kappa c)^{n-q+l-1/2}\cdot K_{n-q-j-1/2(\kappa c)}=B_{n,2,j,l}$$

putting $3/2=l-1/2,\ -3/2=-j-1/2\ \therefore l=2,j=1$

putting $5/2=l-1/2,\ -5/2=-j-1/2\ \therefore l=3,j=2$

$$=\pi/2\cdot[B_{n,2,1,2}/\kappa^3-B_{n,2,2,3}/\kappa^3\cdot\cos^2\gamma]$$

Similarly

$$F_2(\gamma,\kappa,c,n,2)=\pi/2\cdot[B_{n,2,1,2}/\kappa^3-B_{n,2,2,3}/\kappa^3\cdot\sin^2\gamma]$$

Associating them

$$\int_0^\infty\int_0^{\pi/2}[\kappa\cdot F_2(\kappa,\gamma,c,n,2)-\kappa\cdot F_2(\gamma,\kappa,c,n,2)]\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$
$$=\pi/2\int_0^\infty\int_0^{\pi/2}\kappa\cdot[B_{n,2,1,2}+B_{n,2,2,3}\sin^2\gamma\quad+\cos^2\gamma]$$
$$\cdot(1/\kappa^3)\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$
$$=\pi/2\int_0^\infty\int_0^{\pi/2}(\sin^2\gamma-\cos^2\gamma)\cdot B_{n,2,2,3}(1/\kappa^2)\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$
$$=\pi/2\int_0^\infty B_{n,2,2,3}\int_0^{\pi/2}(1-2\cos^2\gamma)/\kappa^2\cdot\cos(\kappa\chi\cos\gamma)\cdot\cos(\kappa y\sin\gamma)d\gamma d\kappa$$

$$*\int_0^{\pi/2}\cos^2\gamma\cdot\cos(a\cos\gamma)\cdot\cos(b\sin\gamma)d\gamma$$
$$=\pi/(2u^2)\cdot[a^2\cdot J_0(u)+(b^2-a^2)/u\cdot J_1(u)]$$

Putting

$a=\kappa\chi$ $\quad b=\kappa y$ and $\quad u=\kappa\cdot\rho$

$$=\pi/(2\kappa^2\rho^2)\cdot[\kappa^2\chi^2\cdot J_0(\kappa\rho)+\kappa^2(y^2-\chi^2)/(\kappa\rho)\cdot J_1(\kappa\rho)]$$
$$=\pi/(2\rho^2)\cdot[\chi^2\cdot J_0(\kappa\rho)+(y^2-\chi^2)/(\kappa\rho)\cdot J_1(\kappa\rho)]$$
$$\int_0^{\pi/2}\cos(a\cos\gamma)\cdot\cos(b\sin\gamma)d\gamma=\pi/2\cdot J_0(\kappa\rho)$$

$$=(\pi/2)^2\int_0^\infty B_{n,2,2,3}[J_0(\kappa\rho)-2/\rho^2\cdot\{\chi^2J_0(\kappa\rho)+(y^2-\chi^2)/(\kappa\rho)\cdot J_1(\kappa\rho)\}]$$
$$\cdot(1/\kappa^2)d\gamma d\kappa$$
$$=\pi^2/4\int_0^\infty B_{n,2,2,3}\cdot 1/(\kappa^2\rho^2)[\rho^2\cdot J_0(\kappa\rho)-2\cdot\{\chi^2J_0(\kappa\rho)+(y^2-\chi^2)/(\kappa\rho)\cdot J_1(\kappa\rho)\}]d\kappa$$
$$\rho^2-2\chi^2=\chi^2+y^2-2\chi^2=y^2-\chi^2$$
$$=\pi^2/4\int_0^\infty B_{n,2,2,3}\cdot(1/\kappa^2\rho^2)(y^2-\chi^2)[J_0(\kappa\rho)-2\cdot J_1(\kappa\rho)/(\kappa\rho)]d\kappa$$

Putting

$$B_2 = \frac{1}{\kappa^2\rho^2}\left[J_0(\kappa\rho) - 2\cdot J_1(\kappa\rho)/(\kappa\rho)\right]$$

$$= \pi^2/4\,\hat{i}\int_0^\infty B_{n,2,2,3}\cdot\left(y^2-\chi^2\right)\cdot B_2 d\kappa$$

****

②−2−2 : from (2, 14, a) and setting $m=0$

$$F_1\ (\kappa,\gamma,c,n,0) = \pi/2\cdot c\cdot\sum_{q=0}^{[n/2]} S_{n,0,q}(c)(\kappa c)^{n-q-1/2} K_{n-q-1/2(\kappa c)}$$

$$= \pi/2\cdot\sum_{q=0}^{[n/2]} S_{n,0,q}(c)\frac{(\kappa c)^{n-q-1/2+1}}{\kappa}\cdot K_{n-q-1/2(\kappa c)}$$

from the definition of c−5

$$\sum_{q=0}^{[n/2]} S_{n,0,q}(c)(\kappa c)^{n-q+1/2}\cdot K_{n-q-1/2(\kappa c)} = B_{n,0,0,1}$$

$$= \pi/2\cdot B_{n,0,0,1}(1/\kappa)$$

hence

The 3rd term $= n(n+1)\int_0^\infty\int_0^{\pi/2}\kappa\cdot(\pi/2)\cdot B_{n,0,0,1}(1/\kappa)\cdot g_1(\kappa,\gamma)d\gamma d\kappa$

$$* \int_0^{\pi/2} g_1(\kappa\gamma)d\gamma d\kappa$$

$$= \int_0^{\pi/2}\cos(\kappa\chi\cos\gamma)\cdot\cos(\kappa y\sin\gamma)d\gamma d\kappa$$

$$= \pi/2\cdot J_0(\kappa\rho)$$

$$= n(n+1)\cdot(\pi/2)^2\int_0^\infty B_{n,0,0,1}\cdot J_0(\kappa\rho)d\kappa$$

Associating these, the coefficients of C_n are

$$G_5(\sigma)\left(-4/\pi^2\right)/2\cdot\left[\pi^4/4\cdot\int_0^\infty B_{n,2,2,3}\left(y^2-\chi^2\right)\cdot B_2 + n(n+1)\cdot B_{n,0,0,1}J_0(\kappa\rho)d\kappa\right]$$

$$= -G_5(\rho)\hat{i}\cdot 1/2\int_0^\infty H_1(c)d\kappa \qquad \rightarrow C_n'$$

****

②・b : About the terms related to B_n $\quad G_5(\sigma)\left(-4/\pi^2\right)/2$

$$\int_0^\infty\int_0^{\pi/2}\kappa\{-F_2(\kappa,\gamma,c,n+1,2)+F_2(\gamma,\kappa,c,n+1,2)+n(n+1)\cdot F_1(\kappa,\gamma,c,n+1,0)\}$$

$$\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$

---

②・b−1 : for F_2 putting in equation (2, 14, b) as $n+1, m=2$

$$F_2(\kappa,\gamma,c,n+1,2) = \pi/2\cdot c^3\left[\sum_{q=0}^{[(n+1)/2]} S_{n+1,2,q}(c)(\kappa c)^{n+1-q-3/2}\cdot K_{n+1-q-3/2(\kappa c)}\right.$$

$$\left. - \sum_{q=0}^{[(n+1)/2]} S_{n+1,2,q}(c)\cdot\kappa^2\cos^2\gamma\cdot c^2(\kappa c)^{n+1-q-5/2}\cdot K_{n+1-q-5/2(\kappa c)}\right]$$

$$= \pi/2\cdot\left[\sum_{q=0}^{[(n+1)/2]} S_{n+1,2,q}(c)\frac{(\kappa c)^{n+1-q-3/2+3}}{\kappa^3}\cdot K_{n-q-1/2(\kappa c)}\right.$$

$$\left. - \sum_{q=0}^{[(n+1)/2]} S_{n+1,2,q}(c)\cdot\kappa^2\cdot c^5(\kappa c)^{n+1-q-5/2}\cdot K_{n-q-3/2(\kappa c)}\cdot\cos^2\gamma\right]$$

$$= \pi/2\cdot\left[\sum_{q=0}^{[(n+1)/2]} S_{n+1,2,q}(c)\frac{(\kappa c)^{n-q+5/2}}{\kappa^3}\cdot K_{n-q-1/2(\kappa c)}\right.$$

$$\left. - \sum_{q=0}^{[(n+1)/2]} S_{n+1,2,q}(c)\cdot\frac{(\kappa c)^{n-q+7/2}}{\kappa^3}\cdot K_{n-q-3/2(\kappa c)}\cdot\cos^2\gamma\right]$$

from the definition of C−5

$$\sum_{q=0}^{[(n+1)/2]} S_{n+1,2,q}(c)(\kappa c)^{n+1-q+l-1/2}\cdot K_{n-q-j+1/2(\kappa c)} = B_{n+1,2,j,l}(c)$$

putting $5/2 = 1+l-1/2,\ -1/2 = 1-j-1/2 \quad \therefore l=2, j=1$

putting $7/2 = l+1/2,\ -3/2 = -j+1/2 \quad \therefore l=3, j=2$

$$= \pi/2\cdot\left[B_{n+1,2,1,2}(c) - B_{n+1,2,2,3}\cdot\cos^2\gamma\right]/\kappa^3$$

By the similar procedure

$$F_2(\gamma,\kappa,c,n+1,2) = \pi/2\cdot\left[B_{n+1,2,1,2}(c) - B_{n+1,2,2,3}(c)\cdot\sin^2\gamma\right]/\kappa^3$$

As a result

$$= \int_0^\infty\int_0^{\pi/2}\kappa\{\pi/2\left(-B_{n+1,2,1,2}(c) + B_{n+1,2,2,3}(c)\cdot\cos^2\gamma\right)$$

$$+\pi/2\left(B_{n+1,2,1,2}(c) - B_{n+1,2,2,3}(c)\cdot\sin^2\gamma\right)\}/\kappa^3\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$

$$= \int_0^\infty\int_0^{\pi/2}(\pi/2)\cdot\left[\cos^2\lambda - \sin^2\gamma\right]\cdot B_{n+1,2,2,3}\,1/\kappa^2\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$

$$= \pi/2\int_0^\infty\int_0^{\pi/2}\left(2\cos^2\gamma - 1\right)\cdot B_{n+1,2,2,3}(c)\cdot g_1(\kappa,\gamma)/\kappa^2\,d\gamma d\kappa$$

since

$$\int_0^{\pi/2}\cos^2\gamma\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$

$$\int_0^{\pi/2}\cos^2\gamma\cdot\cos(\kappa\chi\cos\gamma)\cdot\cos(\kappa y\sin\gamma)d\gamma d\kappa$$

$$= \pi/2\cdot 1/\rho^2\cdot\left[\chi^2\cdot J_0(\kappa\rho) + \left(y^2-\chi^2\right)J_1(\kappa\rho)/(\kappa\rho)\right]$$

$$\int_0^{\pi/2} g_1(\kappa\gamma)d\gamma = \pi/2\cdot J_0(\kappa\rho)$$

$$= (\pi/2)^2\int_0^\infty B_{n+1,2,2,3}(c)\left[2/\rho^2\left\{\chi^2\cdot J_0(\kappa\rho) + \left(y^2-\chi^2\right)\cdot J_1(\kappa\rho)/(\kappa\rho)\right\}/\kappa^2\right.$$

$$\left. - 1/\left(\kappa^2\right)\cdot J_0(\kappa\rho)\right]d\kappa$$

$$= \pi^2/4\int_0^\infty B_{n+1,2,2,3}(c)\cdot 1/\left(\kappa^2\rho^2\right)\left[\left(2\chi^2-\rho^2\right)\cdot J_0(\kappa\rho) + 2\left(y^2-\chi^2\right)\cdot J_1(\kappa\rho)/(\kappa\rho)\right]d\kappa$$

$$-\rho^2 + 2\chi^2 = -\left(\chi^2+y^2\right) + 2\chi^2 = \chi^2 - y^2$$

$$= \pi^2/4\int_0^\infty B_{n+1,2,2,3}(c)\cdot\left(\chi^2-y^2\right)/\left(\kappa^2\rho^2\right)\left[J_0(\kappa\rho) - 2\cdot J_1(\kappa\rho)/(\kappa\rho)\right]d\kappa$$

here putting

$$\frac{1}{\kappa^2\rho^2}\left[J_0(\kappa\rho) - 2\cdot J_1(\kappa\rho)/(\kappa\rho)\right] = B_2$$

$$= \pi^2/4\hat{i}\int_0^\infty B_{n+1,2,2,3}(c)\cdot\left(\chi^2-y^2\right)\cdot B_2 d\kappa$$

****

②・b−2 : for $F_1\ (\kappa,\gamma,c,n+1,0)$, we have from (2, 14, a)

$$F_1\ (\kappa,\gamma,c,n+1,0) = \pi/2\cdot c\sum_{q=0}^{[(n+1)/2]} S_{n+1,0,q}(c)(\kappa c)^{n+1-q-1/2}\cdot K_{n+1-q-1/2(\kappa c)}$$

$$= \pi/2\cdot\sum_{q=0}^{[(n+1)/2]} S_{n+1,0,q}(c)\frac{(\kappa c)^{n-q+3/2}}{\kappa}\cdot K_{n-q+1/2(\kappa c)}$$

from the definition C−5

$$\sum_{q=0}^{[(n+1)/2]} S_{n+1,0,q}(c)(\kappa c)^{n+1-q+l-1/2}\cdot K_{n+1-q-j-1/2(\kappa c)} = B_{n+1,0,j,l}(c)$$

putting $3/2 = 1+l-1/2$ and, $1/2 = 1-j-1/2 \quad \therefore l=1, j=0$

$$= \pi/2\cdot B_{n+1,0,0,1}/\kappa$$

Hence

$$\int_0^\infty\int_0^{\pi/2}\kappa\cdot n(n+1)\cdot\pi/2\cdot B_{n+1,0,0,1}/\kappa\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$

$$= \pi/2\cdot n(n+1)\int_0^\infty\int_0^{\pi/2} B_{n+1,0,0,1}\cdot g_1(\kappa,\gamma)d\gamma d\kappa$$

$$= \pi/2\cdot n(n+1)\int_0^\infty B_{n+1,0,0,1}\cdot\pi/2\cdot J_0(\kappa\rho)d\kappa$$

From these, the coefficient of B_n is

$$(\pi/2)^2\left[\int_0^\infty B_{n+1,2,2,3}(c)\cdot\left(\chi^2-y^2\right)\cdot B_2 + n(n+1)\cdot B_{n+1,0,0,1}\cdot J_0(\kappa\rho)\right]d\kappa$$

$$= (\pi/2)^2\,\hat{i}\int_0^\infty -H_4(c)d\kappa \qquad B_n'$$

****

②・a : for A_n we have $G_5(\sigma)\left(-4/\pi^2\right)$

$$= \int_0^\infty\int_0^{\pi/2}\kappa\cdot g_1(\kappa\gamma)\cdot\{n\cdot(2n-1)\cdot F_2(\kappa,\gamma,c,n,1)\}d\gamma d\kappa +$$

$$+(n-2)/2\int_0^\infty\int_0^{\pi/2}\kappa\cdot g_1(\kappa\gamma)\{F_2(\kappa,\gamma,c,n-1,2) - F_2(\gamma,\kappa,c,n-1,2)\}d\gamma d\kappa$$

$$-n(n+1)(n-2)/2\cdot\int_0^\infty\int_0^{\pi/2}\kappa\cdot g_1(\kappa\gamma)\cdot F_1(\kappa,\gamma,c,n-1,0)d\gamma d\kappa$$

---

②・a−1 : for $F_2(\kappa,\gamma,c,n,1)$ m putting in (2, 14, b) as

$m=1$

$$F_2(\kappa,\gamma,c,n,1)=\pi/2\cdot c^3\left[\sum_{q=0}^{[n/2]}S_{n,1,q}(c)(\kappa c)^{n-q-3/2}\cdot K_{n-q-3/2(\kappa c)}\right.$$
$$\left.-\sum_{q=0}^{[n/2]}S_{n,1,q}(c)\cdot\kappa^2\cos^2\gamma\cdot c^2(\kappa c)^{n-q-5/2}\cdot K_{n-q-5/2(\kappa c)}\right]$$
$$=\pi/2\cdot\left\{c^2\sum_{q=0}^{[n/2]}S_{n,1,q}(c)(\kappa c)^{n-q-3/2+1}/\kappa\cdot K_{n-q-3/2(\kappa c)}\right.$$
$$\left.-c^2\sum_{q=0}^{[n/2]}S_{n,1,q}(c)\kappa^2\cdot\cos^2\gamma\cdot c^3(\kappa c)^{n-q-5/2}\cdot K_{n-q-5/2(\kappa c)}\right\}$$
$$=\pi/2\cdot\left\{c^2\sum_{q=0}^{[n/2]}S_{n,1,q}(c)(\kappa c)^{n-q-1/2}/\kappa\cdot K_{n-q-3/2(\kappa c)}\right.$$
$$\left.-c^2\sum_{q=0}^{[n/2]}S_{n,1,q}(c)\cdot(\kappa c)^{n-q-5/2+3}/\kappa\cdot K_{n-q-5/2(\kappa c)}\cos^2\gamma\right\}$$

from the definition of C−5

$$\sum_{q=0}^{[n/2]}S_{n,1,q}(c)(\kappa c)^{n-q+l-1/2}\cdot K_{n-q-j-1/2(\kappa c)}=B_{n,1,j,l}$$

putting $-1/2=l-1/2,\ -3/2=-j-1/2\quad\therefore l=0,j=1$

putting $1/2=l-1/2,\ -5/2=-j-1/2\quad\therefore l=1,j=2$

$$=\pi/2\cdot c^2/\kappa\left[B_{n,1,1,0}-B_{n,1,2,1}\cos^2\gamma\right]$$

Therefore

$$\int_0^\infty\int_0^{\pi/2}\kappa\cdot g_1(\kappa\gamma)\cdot n(2n-1)\cdot\pi/2\cdot c^2/\kappa\cdot\left(B_{n,1,1,0}-B_{n,1,2,1}\cos^2\gamma\right)d\gamma d\kappa$$
$$=n(2n-1)\cdot\pi/2\cdot c^2\left[\int_0^\infty\int_0^{\pi/2}B_{n,1,1,0}\cdot g_1(\kappa\gamma)d\gamma d\kappa\right.$$
$$\left.-\int_0^\infty\int_0^{\pi/2}B_{n,1,2,1}\cdot g_1(\kappa\gamma)\cos^2\gamma d\gamma d\kappa\right]$$
$$=(\pi/2)\cdot n(2n-1)\cdot c^2\left[\int_0^\infty B_{n,1,1,0}\cdot\pi/2\cdot J_0(\kappa\rho)d\kappa\right.$$
$$\left.-\int_0^\infty B_{n,1,2,1}\cdot\pi/2\cdot1/(\kappa^2\rho^2)\cdot\left[\kappa^2\chi^2\cdot J_0(\kappa\rho)+\frac{(y^2-\chi^2)\kappa^2}{\kappa\cdot\rho}J_1(\kappa\rho)\right]d\kappa\right]$$
$$=(\pi/2)^2\cdot n(2n-1)\cdot c^2\left[\int_0^\infty B_{n,1,1,0}\cdot J_0(\kappa\rho)d\kappa\right.$$
$$\left.-\int_0^\infty B_{n,1,2,1}\cdot1/\rho^2\cdot B_1\,d\kappa\right]$$
$$*\ B_1=\chi^2\cdot J_0(\kappa\rho)+(y^2-\chi^2)/\kappa\rho\cdot J_1(\kappa\rho)$$

****

②・a−2 : for $F_2(\kappa,\gamma,c,n-1,2)$, putting in the equation (2, 14, b)

$n\to n-1,m=2$

$$F_2(\kappa,\gamma,c,n-1,2)=\pi/2\cdot c^3\left[\sum_{q=0}^{[(n-1)/2]}S_{n-1,2,q}(c)(\kappa c)^{n-1-q-3/2}\cdot K_{n-1-q-3/2(\kappa c)}\right.$$
$$\left.-\sum_{q=0}^{[(n-1)/2]}S_{n-1,2,q}(c)\cdot\kappa^2\cos^2\gamma\cdot c^2(\kappa c)^{n-1-q-5/2}\cdot K_{n-1-q-5/2(\kappa c)}\right]$$
$$=\pi/2\cdot\left[\sum_{q=0}^{[(n-1)/2]}S_{n-1,2,q}(c)(\kappa c)^{n-1-q-3/2+3}/\kappa^3\cdot K_{n-q-5/2(\kappa c)}\right.$$
$$\left.-\sum_{q=0}^{[(n-1)/2]}S_{n-1,2,q}(c)\cdot(\kappa c)^{n-1-q-5/2+5}/\kappa^3\cdot K_{n-1-q-5/2(\kappa c)}\cdot\cos^2\gamma\right]$$

from the definition of C−5

$$\sum_{q=0}^{[(n-1)/2]}S_{n-1,2,q}(c)(\kappa c)^{n-1-q+l-1/2}\cdot K_{n-1-q-j-1/2}=B_{n-1,2,j,l}$$

putting $-1-3/2+3=l-3/2,\ -5/2=-1-j-1/2\quad\therefore l=2,j=1$

putting $-1-5/2+5=l-3/2,\ -1/2=-j-3/2\quad\therefore l=3,j=2$

$$=\pi/2\cdot\left[B_{n-1,2,1,2}-B_{n-1,2,2,3}\cdot\cos^2\gamma\right]/\kappa^3$$

By the similar procedure

$$F_2(\gamma,\kappa,c,n-1,2)=\pi/2\cdot\left[B_{n-1,2,1,2}-B_{n-1,2,2,3}\cdot\sin^2\gamma\right]/\kappa^3$$

Associating these

$$(n-2)/2\int_0^\infty\int_0^{\pi/2}\kappa\cdot g_1(\kappa\gamma)\cdot\pi/2/\kappa^3\cdot d\gamma d\kappa$$
$$\left[B_{n-1,2,1,2}-B_{n-1,2,2,3}\cos^2\gamma-\left(B_{n-1,2,1,2}-B_{n-1,2,2,3}\sin^2\gamma\right)\right]d\gamma d\kappa$$
$$=(n-2)/2\cdot\pi/2\int_0^\infty\int_0^{\pi/2}\cdot g_1(\kappa\gamma)/\kappa^2\left(\sin^2\gamma-\cos^2\gamma\right)\cdot B_{n-1,2,2,3}d\gamma d\kappa$$
$$=\frac{(n-2)/2\cdot\pi/2}{\kappa^2}\cdot\left[\int_0^\infty\int_0^{\pi/2}\cos(\kappa\chi\cos\gamma)\cos(\kappa y\sin\gamma)\cdot B_{n-1,2,2,3}d\gamma d\kappa\right.$$
$$\left.-2\int_0^\infty\int_0^{\pi/2}\cos^2(\gamma)\cos(\kappa\chi\cos\gamma)\cos(\kappa y\sin\gamma)\cdot B_{n-1,2,2,3}\,d\gamma d\kappa\right]$$
$$=(n-2)/2\cdot\pi/2/\kappa^2\cdot\left[\int_0^\infty B_{n-1,2,2,3}\cdot(\pi/2)\cdot J_0(\kappa\rho)d\kappa\right.$$
$$\left.-2\cdot\pi/2\int_0^\infty1/(\kappa^2\rho^2)\left(\kappa^2\chi^2J_0(\kappa\rho)+\kappa^2(y^2-\chi^2)\cdot J_1(\kappa\rho)/(\kappa\rho)\right)B_{n-1,2,2,3}\,d\kappa\right]$$
$$=(\pi/2)^2(n-2)/2/\kappa^2\cdot\left[\int_0^\infty B_{n-1,2,2,3}\cdot\left\{1/\rho^2\right\}\right.$$
$$\left.\left\{J_0(\kappa\rho)-2\left(\chi^2\cdot J_0(\kappa\rho)+(y^2-\chi^2)\cdot J_1(\kappa\rho)/(\kappa\rho)\right)\right\}d\kappa\right]$$
$$\rho^2-2\chi^2=\chi^2+y^2-2\chi^2=y^2-\chi^2$$
$$=(\pi/2)^2(n-2)/2/\kappa\cdot1/(\kappa^2\rho^2)\left[\int_0^\infty B_{n-1,2,2,3}(y^2-\chi^2)\left[J_0(\kappa\rho)-2J_1(\kappa\rho)/(\kappa\rho)\right]d\kappa\right]$$

here, putting

$$B_2=1/(\kappa^2\rho^2)\left[J_0(\kappa\rho)-2\cdot J_1(\kappa\rho)/(\kappa\rho)\right]$$
$$=(\pi/2)^2(n-2)/2\cdot(y^2-\chi^2)\int_0^\infty B_2\cdot B_{n-1,2,2,3}d\kappa$$

****

②・a−3 : for $F_2(\kappa,\gamma,c,n-1,0)$, from the equation (2, 14, a)

$$F_1(\kappa,\gamma,c,n-1,0)=\pi/2\cdot c\sum_{q=0}^{[(n-1)/2]}S_{n-1,0,q}(c)(\kappa c)^{n-1-q-1/2}\cdot K_{n-1-q-1/2(\kappa c)}$$
$$=\pi/2\cdot\sum_{q=0}^{[(n-1)/2]}S_{n-1,0,q}(c)\frac{(\kappa c)^{n-q-3/2+1}}{\kappa}\cdot K_{n-q-3/2(\kappa c)}$$

from the definition of C−5

$$\sum_{q=0}^{[(n-1)/2]}S_{n-1,0,q}(c)(\kappa c)^{n-1-q+l-1/2}\cdot K_{n-1-q-j-1/2}=B_{n-1,0,j,l}$$

putting $-1/2=l-3/2,\ \therefore l=1,-3/2=-3/2-j\quad\therefore j=0$

hence $F_1(\kappa,\gamma,c,n-1,0)=\pi/2\cdot B_{n-1,0,0,1}/\kappa$

Therefore, we have

$$-n(n+1)(n-2)/2\int_0^\infty\int_0^{\pi/2}\kappa\cdot g_1(\kappa\gamma)\cdot\pi/2\cdot B_{n-1,0,0,1}/\kappa\,d\gamma d\kappa$$
$$=-\pi/2\cdot n(n+1)(n-2)/2\int_0^\infty\int_0^{\pi/2}g_1(\kappa\gamma)\cdot B_{n-1,0,0,1}d\gamma d\kappa$$
$$=-(\pi/2)^2\cdot n(n+1)(n-2)/2\int_0^\infty B_{n-1,0,0,1}\cdot J_0(\kappa\rho)d\kappa$$

Associating these, about the A_n

$$(\pi/2)^2\left[n(2n-1)c^2\left\{\int_0^\infty B_{n,1,1,0}\cdot J_0(\kappa\rho)-\int_0^\infty B_{n,1,2,1}B_1/\rho^2\right\}\right.$$
$$+(n-2)/2\cdot(y^2-\chi^2)\int_0^\infty B_{n-1,2,2,3}\cdot B_2$$
$$\left.-n(n+1)(n-2)/2\int_0^\infty B_{n-1,0,0,1}\cdot J_0(\kappa\rho)\right]d\kappa$$
$$=(\pi/2)^2\hat{i}(-H_1(c))\quad\text{components of}\to A_n'\quad C_{1a}$$

The An, Bn and Cn functions in (2.16)

$$A_n^*(\alpha,\beta,z_i)=-\frac{4}{\pi^2}\left\{n(2n-1)F_2(\alpha,\beta,z_i,n,1)+\frac{1}{2}(n-2)\right.$$
$$\times\left[F_2(\alpha,\beta,z_i,n-1,2)-F_2(\beta,\alpha,z_i,n-1,2)\right]\text{-------(A-1)}$$
$$\left.-\frac{1}{2}n(n+1)(n-2)F_1(\alpha,\beta,z_i,n-1,0)\right\}$$
$$B_n^*(\alpha,\beta,z_i)=-\frac{4}{\pi^2}\left\{-\frac{1}{2}\left[F_2(\alpha,\beta,z_in+1,2)-F_2(\beta,\alpha,z_i,n+1,2)\right]\right.\quad\text{---(A2)}$$
$$\left.+\frac{1}{2}n(n+1)F_1(\alpha,\beta,z_i,n+1,0)\right\}$$
$$C_n^*(\alpha,\beta,z_i)=-\frac{4}{\pi^2}\left\{\frac{1}{2}\left[F_2(\alpha,\beta,z_in,2)-F_2(\beta,\alpha,z_i,n,2)\right]+\frac{1}{2}n(n+1)F_1(\alpha,\beta,z_i,n,0)\right\}$$

$$A_n^{**}(\alpha,\beta,z_i) = -\frac{4}{\pi^2}\left[n(2n-1)F_3(\alpha,\beta,z_i n,1)+(n-2)F_3(\alpha,\beta,z_i,n-1,2)\right] \quad \text{(A-4)}$$

$$B_n^{**}(\alpha,\beta,z_i) = \frac{4}{\pi^2}F_3(\alpha,\beta,z_i,n+1,2), \quad \text{(A-5)}$$

$$C_n^{**}(\alpha,\beta,z_i) = -\frac{4}{\pi^2}F_3(\alpha,\beta,z_i,n,2); \quad \text{(A-6)}$$

$$A_n^{***}(\alpha,\beta,z_i) = -\frac{4}{\pi^2}\left[n(2n-1)z_iF_4(\alpha,\beta,z_i n,1)-(n+1)(n-2)F_4(\alpha,\beta,z_i,n-1,1)\right]$$

$$B_n^{***}(\alpha,\beta,z_i) = \frac{4}{\pi^2}nF_4(\alpha,\beta,z_i,n+1,1), \quad \text{(A-8)}$$

$$C_n^{***}(\alpha,\beta,z_i) = \frac{4}{\pi^2}F_4(\alpha,\beta,z_i,n,1). \quad \text{(A-9)}$$

The inner set of integrals required by (2,22). u = √(a*a + b*b),

J0 and J1 are Bessel function of the first and the second kind.

$$\int_0^{\frac{1}{2}\pi}\cos(a\cos\gamma)\cos(b\sin\gamma)d\gamma = \frac{\pi}{2}J_0(u), \quad \text{(B 1)}$$

$$\int_0^{\frac{1}{2}\pi}\cos^2\gamma\cos(a\cos\gamma)\cos(b\sin\gamma)d\gamma = \frac{\pi}{2u^2}\left[a^2J_0(u)+\frac{b^2-a^2}{u}J_1(u)\right], \quad \text{(B 2)}$$

$$\int_0^{\frac{1}{2}\pi}\cos\gamma\sin\gamma\sin(a\cos\gamma)\sin(b\sin\gamma)d\gamma = -\frac{\pi}{2}\frac{ab}{u^2}\left[J_0(u)-\frac{2}{u}J_1(u)\right], \quad \text{(B 3)}$$

$$\int_0^{\frac{1}{2}\pi}\cos\gamma\sin(a\cos\gamma)\cos(b\sin\gamma)d\gamma = \frac{\pi}{2}\frac{a}{u}J_1(u). \quad \text{(B 4)}$$

The primed An', Bn' and Cn' contained in (2-24)

$$A_n' = \int_0^\infty\{G_5(\eta)H_1(-b)-G_5(\sigma)H_1(c)+G_6(\sigma,\eta)H_2(-b)-G_6(\eta,\sigma)H_2(c)$$
$$+G_1(\sigma,\eta)H_3(-b)-G_1(\eta,\sigma)H_3(c)\}d\kappa \quad \text{(C 1a)}$$

$$B_n' = \int_0^\infty\{G_5(\eta)H_4(-b)-G_5(\sigma)H_4(c)+G_6(\sigma,\eta)H_5(-b)-G_6(\eta,\sigma)H_5(c)$$
$$+G_1(\sigma,\eta)H_6(-b)-G_1(\eta,\sigma)H_6(c)\}d\kappa \quad \text{(C 1b)}$$

$$C_n' = \int_0^\infty\{G_5(\eta)H_7(-b)-G_5(\sigma)H_7(c)+G_6(\sigma,\eta)H_8(-b)-G_6(\eta,\sigma)H_8(c)$$
$$+G_1(\sigma,\eta)H_9(-b)-G_1(\eta,\sigma)H_9(c)\}d\kappa \quad \text{(C 1c)}$$

****

$$A_n'' = \int_0^\infty\{G_6(\sigma,\eta)H_{10}(-b)-G_6(\eta,\sigma)H_{10}(c)$$
$$+G_1(\sigma,\eta)H_{12}(-b)-G_1(\eta,\sigma)H_{12}(c)$$
$$+G_3(\sigma,\eta)H_{11}(-b)-G_3(\eta,\sigma)H_{11}(c)\}d\kappa \quad \text{(C 1d)}$$

$$B_n'' = \int_0^\infty\{G_6(\sigma,\eta)H_{13}(-b)-G_6(\eta,\sigma)H_{13}(c)$$
$$+G_1(\sigma,\eta)H_{15}(-b)-G_1(\eta,\sigma)H_{15}(c)$$
$$+G_3(\sigma,\eta)H_{14}(-b)-G_3(\eta,\sigma)H_{14}(c)\}d\kappa \quad \text{(C 1e)}$$

$$C_n'' = \int_0^\infty\{G_6(\sigma,\eta)H_{16}(-b)-G_6(\eta,\sigma)H_{16}(c)$$
$$+G_1(\sigma,\eta)H_{18}(-b)-G_1(\eta,\sigma)H_{18}(c)$$
$$+G_3(\sigma,\eta)H_{17}(-b)-G_3(\eta,\sigma)H_{17}(c)\}d\kappa \quad \text{(C 1f)}$$

$$A_n''' = \int_0^\infty\{G_2(\sigma,\eta)H_{19}(-b)-G_2(\eta,\sigma)H_{19}(c)$$
$$+G_4(\sigma,\eta)H_{20}(-b)-G_4(\eta,\sigma)H_{20}(c)\}d\kappa, \quad \text{(C 1g)}$$

$$B_n''' = \int_0^\infty\{G_2(\sigma,\eta)H_{21}(-b)-G_2(\eta,\sigma)H_{21}(c)+G_4(\sigma,\eta)H_{22}(-b)$$
$$-G_4(\eta,\sigma)H_{22}(c)\}d\kappa, \quad \text{(C 1h)}$$

$$C_n''' = \int_0^\infty\{G_2(\sigma,\eta)H_{23}(-b)-G_2(\eta,\sigma)H_{23}(c)+G_4(\sigma,\eta)H_{24}(-b)$$
$$-G_4(\eta,\sigma)H_{24}(c)\}d\kappa, \quad \text{(C 1i)}$$

Where

$$H_1(z_i) = -n(2n-1)z_i^2J_0(\kappa\rho)B_{n,1,1,0}(z_i)+n(2n-1)\frac{z_i^2}{\rho^2}B_1B_{n,1,2,1}(z_i)$$
$$-\frac{1}{2}(n-2)(y^2-x^2)B_2B_{n-1,2,2,3}(z_i)+\frac{1}{2}n(n+1)(n-2)J_0(\kappa\rho)B_{n-1,0,0,1}(z_i) \quad \text{(C 2a)}$$

$$H_2(z_i) = \left\{-n(2n-1)z_i^2\left[B_{n,1,1,0}(z_i)-B_{n,1,2,1}(z_i)\right]+\frac{1}{2}(n-2)z_i^2B_{n-1,2,2,1}(z_i)\right.$$
$$\left.+\frac{1}{2}n(n+1)(n-2)B_{n-1,0,0,1}(z_i)\right\}\frac{B_1}{\rho^2} \quad \text{(C 2b)}$$

$$H_3(z_i) = \kappa z_i^2\left[-n(2n-1)z_iB_{n,1,1,0}(z_i)+(n+1)(n-2)B_{n-1,1,1,0}(z_i)\right]\frac{B_1}{\rho^2} \quad \text{(C 2c)}$$

$$H_4(z_i) = \frac{1}{2}\left[(y^2-x^2)B_2B_{n+1,2,2,3}(z_i)-n(n+1)J_0(\kappa\rho)B_{n+1,0,0,1}(z_i)\right] \quad \text{(C 2d)}$$

$$H_5(z_i) = -\frac{1}{2}\left[z_i^2B_{n+1,2,2,1}(z_i)+n(n+1)B_{n+1,0,0,1}(z_i)\right]\frac{B_1}{\rho^2} \quad \text{(C 2e)}$$

$$H_6(z_i) = n\frac{\kappa z_i^2}{\rho^2}B_1B_{n+1,1,1,0}(z_i) \quad \text{(C 2f)}$$

$$H_7(z_i) = -\frac{1}{2}\left[(y^2-x^2)B_2B_{n,2,2,3}(z_i)+n(n+1)J_0(\kappa\rho)B_{n,0,0,1}(z_i)\right] \quad \text{(C 2g)}$$

$$H_8(z_i) = \frac{1}{2}\left[z_i^2B_{n,2,2,1}(z_i)-n(n+1)B_{n,0,0,1}(z_i)\right]\frac{B_1}{\rho^2} \quad \text{(C 2h)}$$

$$H_9(z_i) = \kappa z_i^2B_{n,1,1,0}(z_i)\frac{B_1}{\rho^2} \quad \text{(C 2i)}$$

$$H_{10}(z_i) = xyB_2\left[-n(2n-1)B_{n,1,1,2}(z_i)-\frac{1}{2}(n-2)B_{n-1,2,2,3}(z_i)\right.$$
$$\left.+\frac{1}{2z_i^2}n(n+1)(n-2)B_{n-1,0,0,3}(z_i)\right] \quad \text{(C 2j)}$$

$$H_{11}(z_i) = xyB_2\left[n(2n-1)B_{n,1,2,3}(z_i)+(n-2)B_{n-1,2,2,3}(z_i)\right] \quad \text{(C 2k)}$$

$$H_{12}(z_i) = \kappa xyB_2\left[-n(2n-1)z_iB_{n,1,1,2}(z_i)+(n+1)(n-2)B_{n-1,1,1,2}(z_i)\right] \quad \text{(C 2l)}$$

$$H_{13}(z_i) = \frac{1}{2}xyB_2\left[B_{n+1,2,2,3}(z_i)-n(n+1)\frac{1}{z_i^2}B_{n+1,0,0,3}(z_i)\right] \quad \text{(C 2m)}$$

$$H_{14}(z_i) = -xyB_2B_{n+1,2,2,3}(z_i) \quad \text{(C 2n)}$$

$$H_{15}(z_i) = n\kappa xyB_2B_{n+1,1,1,2}(z_i) \quad \text{(C 2o)}$$

$$H_{16}(z_i) = -\frac{1}{2}xyB_2\left[B_{n,2,2,3}(z_i)+n(n+1)\frac{1}{z_i^2}B_{n,0,0,3}(z_i)\right] \quad \text{(C 2p)}$$

$$H_{17}(z_i) = xyB_2B_{n,2,2,3}(z_i) \quad \text{(C 2q)}$$

$$H_{18}(z_i) = \kappa xyB_2B_{n,1,1,2}(z_i) \quad \text{(C 2r)}$$

$$H_{19}(z_i) = -x\frac{J_1(\kappa\rho)}{\kappa^2\rho}\left\{n(2n-1)\left[B_{n,1,1,2}(z_i)-B_{n,1,2,3}(z_i)\right]\right.$$
$$\left.-\frac{1}{2}(n-2)B_{n-1,2,2,3}(z_i)-\frac{1}{2}n(n+1)(n-2)\frac{1}{z_i^2}B_{n-1,0,0,3}(z_i)\right\} \quad \text{(C 2s)}$$

$$H_{20}(z_i) = -x\frac{J_1(\kappa\rho)}{\kappa\rho}\left[n(2n-1)z_iB_{n,1,1,2}(z_i)-(n+1)(n-2)B_{n-1,1,1,2}(z_i)\right] \quad \text{(C 2t)}$$

$$H_{21}(z_i) = -\frac{1}{2}x\frac{J_1(\kappa\rho)}{\kappa^2\rho}\left[B_{n+1,2,2,3}(z_i)+n(n+1)\frac{1}{z_i^2}B_{n+1,0,0,3}(z_i)\right] \quad \text{(C 2u)}$$

$$H_{22}(z_i) = nx\frac{J_1(\kappa\rho)}{\kappa\rho}B_{n+1,1,1,2}(z_i) \quad \text{(C 2v)}$$

$$H_{23}(z_i) = \frac{1}{2}x\frac{J_1(\kappa\rho)}{\kappa^2\rho}\left[B_{n,2,2,3}(z_i)-n(n+1)\frac{1}{z_i^2}B_{n,0,0,3}(z_i)\right] \quad \text{(C 2w)}$$

$$H_{24}(z_i) = x\frac{J_1(\kappa\rho)}{\kappa\rho}B_{n,1,1,2}(z_i) \quad \text{(C 2x)}$$

where

$$B_1 = x^2J_0(\kappa\rho)+(y^2-x^2)\frac{J_1(\kappa\rho)}{\kappa\rho} \quad \text{(C 3a)}$$

$$B_2 = \frac{1}{\kappa^2\rho^2}\left[J_0(\kappa\rho)-2\frac{J_1(\kappa\rho)}{\kappa\rho}\right] \quad \text{(C 3b)}$$

$$\rho = (x^2+y^2)^{\frac{1}{2}} \quad \text{(C 4)}$$

$$B_{n,m,j,l}(z_i) = \sum_{q=0}^{\left[\frac{1}{2}n\right]}S_{nmq}(z_i)(\kappa|z_i|)^{n-q+l-\frac{1}{2}}K_{n-q-j-\frac{1}{2}}(\kappa|z_i|) \quad \text{(C 5)}$$